Laboratory simulation of belt press dewatering: Application of the Darcy equation to gravity drainage

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ABSTRACT: Belt-thickening processes are used for preliminary dewatering of flocculent sludges such as polymer-conditioned wastewater sludges, industrial sludges, and water treatment silts. These processes are also directly or indirectly used as the lead-in step in belt-pressing dewatering of these types of sludges. In the belt press application, improperly drained sludge can yield failure of the subsequent pressing steps. This paper is Part I of a series on the laboratory simulation of belt-pressing dynamics and addresses the mathematics of the gravity drainage of conditioned sludges through short depths on fabric filter cloth. The rate of water drainage through the fabric filter for sludge from a given source and treated with constant polymer doses per dry weight of sludge solids is a function of the total volume of the sample, the solids fraction within the sample, the specific resistance of the forming cake, and the specific resistance of the filter cloth. A drainage rate model that fits the general shape of drainage rate from numerous sludges is derived. Methods for using the proper test volume in batch tests in relation to full-scale dynamic presses are presented. Finally, a simple test method for determining the resistance through the filter cloth on full-scale presses is suggested. Data from 12 test conditions including five types of sludges are analyzed. The model appears to adequately account for volumetric changes, sludge concentration changes, and changes in belt resistance in laboratory drainage tests. *Water Environ. Res.,* **68,** 359 (1996).

KEYWORDS: Darcy equation, dewatering, filter press, polymer, sludge, specific resistance.

Gravity belt thickening of conditioned sludges is commonly practiced as an individual unit process and as an integral pretreatment for pressure dewatering on belt filter presses. The quality and quantity of sludge produced by the gravity drainage section of a belt press controls the performance of the pressure end of the press. Although laboratory-scale methods for simulating drainage rates and conditioning regimes have been developed (Poduska and Collins, 1980; Baskerville *et al.,* 1978), the relationship of the derived rates has been limited to empirical correlations to press operation. A mathematical analysis of the gravity drainage phenomenon has not been developed sufficiently to answer fundamental questions of efficiency and design. It is the overall purpose of this study to produce a system for predicting the performance of the belt press process from laboratory data. The focus of this paper is to define the dynamics of the gravity drainage section of the belt press process. The information presented herein is also applicable to the gravity drainage belts as stand-alone processes.

Darcy's equation, which models flow through porous media, has been used successfully for estimation of flow in complex media such as soil (Freeze and Cherry, 1979), rotary vacuum filters (Coackley and Jones, 1956), plate and frame filters (Ruth *et al.,* 1933; Ruth, 1935), and sand filters (Carman, 1933). Much of the pioneering work in dewatering phenomenon evolved around high-pressure systems, such as plate and frame presses or vacuum filters. The constraint imposed on the mathematical systems associated with these process models was the assumption of constant pressure over time. When the mathematical system also modeled the formation of a cake, it was argued that the specific resistance of the forming cake was constant for noncompressible materials or that an average resistance over the depth of cake could be used (Ruth, 1946) even if the sludge was compressible (Underwood, 1928). However, Ruth's filter resistance has been challenged. Tiller and Shirato (1964) and others suggested that the flow resistance of compressible sludges should be modeled as applied pressure to an empirical power (Coackley and Jones, 1956; Svarovsky, 1979). In these high-pressure systems, the cake first undergoes filtration and then consolidation. This concept was developed into a general consolidation theory (Shirato *et al.,* 1985). Kos and Adrian (1975) pointed out that wastewater sludge is always compressible because of the flocculent nature of the material. These authors also pointed out that the concentration of suspended solids in the cake varies significantly as a function of the intraparticle pressures within the cake.

Fundamental differences exist between the high-pressure models discussed above and gravity belt drainage. The maximum deformation of the cake during gravity drainage is created by the original depth of sludge applied to the belt, which is usually less than 6 cm of water pressure. It is reasonably expected that an average specific resistance, such as presented by the works of Ruth (1946) and Underwood (1928), will suffice to describe the cake resistance. Furthermore, the limited pressure applied to the cake is solely due to the available head in the slurry, which diminishes rapidly with time. Therefore, the existing filtration models do not adequately describe gravity drainage under low pressure and a need exists to explore the dynamics of low-pressure systems.

The derivation of a drainage rate equation for gravity belt drainage is described in this paper. Substantiating data collected from sludge drainage tests using a variety of industrial and municipal sludges are presented along with a systematic method for data reduction. The utility of the equation as a tool for understanding filter press operation is explored.

Role of Gravity Drainage in the Belt Filter Process

The belt press filter is a complex physical-chemical process, conceptualized as occurring in four zones as depicted in Figure

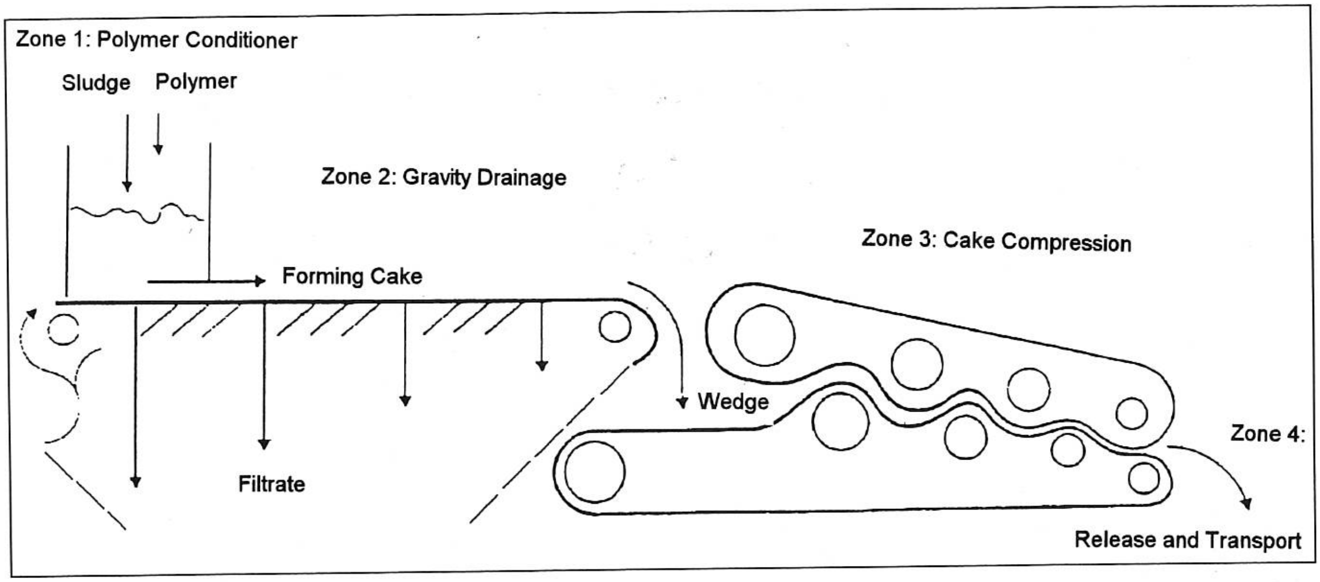

Figure 1—Schematic of belt press.

1 (Haworth, 1973; U.S. EPA, 1987). Belt presses may be the two-belt variety in which the lower pressure belt is used as the gravity drainage section or three-belt presses where an independent belt is used for gravity drainage. In Zone 1, sludge is conditioned to produce a drainable floc. The conditioned sludge is then applied to a gravity drainage belt (Zone 2) where water is removed from the forming cake. For wastewater sludges, it is typical for one- to two-thirds of the water to be removed in this step of the process (Haworth, 1973). Drained cake then enters the press zone (Zone 3) where a top belt is contacted with the bottom belt, forming a low-pressure wedge zone. A series of rollers then contact the belts under increasingly higher pressures. Finished cake is released from the belt in Zone 4.

The first point of process failure (Class 1 or low-pressure failure) occurs when too much sludge, improperly conditioned sludge, or insufficiently drained sludge, enters the low-pressure wedge zone. The improperly conditioned sludge spills out between the belts (wet migration). Although the press may continue to produce cake, the cake near the edges of the belt is wet, the capture efficiency of the machine decreases, and housekeeping problems arise. The second point of failure (Class 2 or hard migration) occurs midway through the first or second set of high-pressure rollers. Partially pressed sludge is squeezed between the belts and "migrates" off the belts. The third failure point (Class 3) is extrusion of sludge into the weave of the belt. The final point of process failure (Class 4) is caused by poor release of the cake from the belt. Class 3 or Class 4 failures create dirty belts that reduce the ability of the belt to pass water and, therefore, increases the likelihood of Class 1 or Class 2 failures.

The overall purpose of this project is to establish a method of analysis of belt press performance by examining the root causes of process failure. Laboratory methods and experimental protocol are needed to diagnose the potential for failure. Predictive information must be translatable to better process design and operations. This paper focuses on defining gravity drainage kinetics and how the operational parameters of belt speed and solids loading affect the quantity and quality of the sludge entering the low-pressure wedge zone.

Derivation of the Belt Press Drainage Equation

In its most general form, Darcy's law equates the rate of fluid flow through a porous medium, dV_f/dt, to the available pressure, P, area, A, resistance, R, viscosity, μ, and the depth of the porous medium, L. For stratified media, such as a sludge cake on top of a filter cloth, it is common to distribute the resistance serially as in Equation 1, where r and ℓ are the resistance and depth of the filter cloth, respectively.

$$\frac{dV_f}{dt} = \frac{PA}{R\mu L + r\mu\ell} \tag{1}$$

For the gravity drainage test described in Test Methods and the drainage on a belt press without plows, the pressure available to drive water through the resistant layers is solely due to gravity. The effective pressure P is related to the effective weight of the slurry, h_e.

$$P = \rho g h_e \tag{2}$$

The density of the slurry above the forming cake, ρ, is assumed to be constant with time, and the contribution of water within the cake on the overall driving force is assumed to be negligible. The effective height is then related to known parameters. In Equation 3, V_o is the initial measured slurry volume, V_f is the measured filtrate volume at any time during the test, and V_c is the calculated volume of the formed cake at time t.

$$h_e = (V_o - V_f - V_c)/A \tag{3}$$

In Equation 3 it is assumed that volume is conserved, that is, the cake is inelastic. This is a fair approximation because pressures are low, and any deformation of the sludge volume will result in compression of the cake and the further release of filtrate. Kos and Adrian (1975) noted that all wastewater sludges appear to be inelastic and flocculent in nature and define such behavior as being consistent with conservation of volume. The depth of cake, L, at any time is therefore related to the volume of the cake deposited at any time, V_c, over the filter area A.

$$L = V_c/A \tag{4}$$

A separation coefficient, S, is introduced to relate the final distribution of volumes between cake and filtrate. The value of S is the ratio of the cake volume to the filtrate volume, V_F, after an infinite drainage time. By the assumption of conserved volumes, S is also equal to the ratio of volume of cake to the filtrate at any time during the test. This concept was originally adopted by Underwood (1928) and is expressed as Equation 5.

$$S = \frac{V_\infty}{V_F} = \frac{V_c}{V_f} \tag{5}$$

When specific resistance is developed in units of length per unit mass, and L is correspondingly developed as mass per area, as in most models for high-pressure filtration, the solids content within the cake must be corrected for volume changes (Gale, 1967; Svarovsky, 1979). This is unnecessary in the present derivation because forces on the cake are small, the change in density is small, and the units of measure of specific resistance expressed in terms of length and volume.

The differential equation (Equation 6) relating the time variable pressure and time variable cake depth to the change in filtrate volume with respect to time, is found by substitution of Equations 2, 3, 4, and 5 into Equation 1.

$$\frac{dV_f}{dt} = \left(\frac{\rho g}{\mu}\right) \frac{V_o - (1 + S)V_f}{(SRV_f/A) + r\ell} \tag{6}$$

Equation 6 is rearranged and separated for integration over the limits $V_f = 0$ at $t = 0$, as in Equation 7. The solution is given as Equation 8.

$$\frac{SR}{A} \int \frac{V_f}{V_o - (1 + S)V_f} dV_f + r\ell \int \frac{1}{V_o - (1 + S)V_f} dV_f = \frac{\rho g}{\mu} \int dt \tag{7}$$

$$\frac{SR}{A(1 + S)^2} \left(-V_o \ln\left(\frac{V_o - (1 + S)V_f}{V_o}\right) - (1 + S)V_f\right) - \frac{r\ell}{(1 + S)} \ln\left(\frac{V_o - (1 + S)V_f}{V_o}\right) = \frac{\rho g}{\mu} t \tag{8}$$

Equation 8 may be rewritten in dimensionless terms as Equation 9 where V_f/V_F is the fraction of total filtrate collected at time t. The left hand term, $KABt$, is a lumped dimensionless time factor. The other terms found in Equation 9, K, B, and γ are defined in Equations 10 to 12. The definition of κ is given in Equation 13.

$$KABt = -\frac{V_f}{V_F} - \ln\left(1 - \frac{V_f}{V_F}\right) - \gamma \ln\left(1 - \frac{V_f}{V_F}\right) \tag{9}$$

Based on Equation 9, it can be implied that the fraction of filtrate recovered, V_f/V_F, is a function of dimensionless time, $KABt$, and the relative resistance of the cake and cloth, γ. Figure 2 is an example of this simple relationship. If γ is small, the cake resistance is much greater than the cloth resistance, and the curves in this figure were generated on the basis of γ values ranging from 0 to 0.8. It is evident from this figure that differences in γ of less than 0.1 between any two test runs will not be readily discernible in the test results.

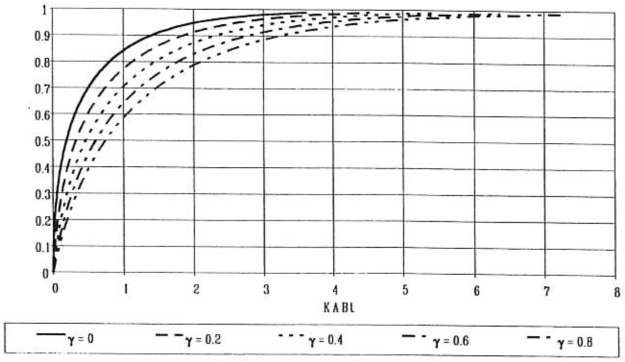

Figure 2—Fraction filtrate recovery as a function of *KABt* and γ. Ordinate is fraction filtrate volume, V_f/V_F.

$$K = \left(\frac{\rho g}{\mu R}\right) = \text{cake permeability factor, cm/}s \tag{10}$$

$$B = \left(\frac{V_F + V_\infty}{V_F \cdot V_\infty}\right) = \text{belt loading fator, } 1/cm^3 \tag{11}$$

$$\gamma = \left(\frac{KA\ell}{V_\infty \kappa}\right) = \text{resistance ratio (dimensionless)} \tag{12}$$

$$\kappa = \left(\frac{\rho g}{\mu r}\right) = \text{filter cloth permeability factor, cm/}s \tag{13}$$

Determination of Filter Cloth Permeability, κ/ℓ

Resistance to flow through the filter cloth is usually less than the resistance through the cake; however, with tightly woven, aged, or dirty cloth, the cloth may represent a significant resistance to drainage. Because of the variability of data from sludge drainage tests, it is difficult to back calculate both κ and K from the sludge drainage data. It is recommended that κ/ℓ be determined independently. In a small test system, it is possible to apply a steady stream of water, Q, across a cloth of known area, A. In this test, $Q = dV_f/dt$ and the available head, h_o, are constant. The value of κ/ℓ is calculated as in Equation 14 (Kos and Adrian, 1975). Values of κ/ℓ for the filter cloth used in this study were measured by flowing water at constant rates into short stand pipes held tightly onto pieces of filter cloth. Values of κ/ℓ ranged from 0.07 to 5.64, depending on the weave of the material (Table 1).

$$\frac{\kappa}{\ell} = \frac{Q}{Ah_o} \tag{14}$$

A similar method has been proposed as a means of judging the cleanliness of belts on operating presses where the rate of water addition is measured for a constant 1.2 cm water height in the stand pipe (Land, 1992). The use of this method is based on the assumption that the filtrate behaves similarly to water. Karr and Keinath (1978) point out that sludges with superfine colloidal particles will affect the apparent resistance of the filter cloth by blinding of the cloth. This phenomenon has not been studied in this laboratory. This effect is not included in the developed equations.

Determination of Sample Size

A critical operations parameter is the quantity of sludge to be treated for a given belt press design. This information is

Table 1—Clean water steady-state evaluation of κ/ℓ.

Belt type	Test area, cm^2	Flow, cm^3/s	Head, cm	κ/ℓ, s^{-1}
Scandiafelt 3366, new	9.1	230	4.5	5.64
	15.9	178	2.0	5.59
	15.9	348	4.0	5.47
IFC 6461, new	9.1	49	4.0	1.34
Fabric for Industry 110-50, used, dirty	15.9	52.6	7.2	0.46
	15.9	27.7	3.1	0.56
Scandiafelt 4570, new	15.9	8	2.0	0.26
	15.9	48	10.0	0.30
GK-DUSA 250 CFM, new	15.9	6.7	5.2	0.08
	15.9	8.3	7.5	0.07

usually based on the experiences of the belt press manufacturer or from empirical databases (ASCE, 1988). A laboratory method should also be able to provide data to confirm the design range given by the belt press manufacturer. To do this, the proper range of test sample volumes must be targeted, matching the batch sample size to a projected loading on a dynamic system. The batch sample size, V_o, is a function of the target belt speed, s_b, belt press width, W, sludge flow, Q_s, polymer flow, Q_p, and test filter area, A. The target sample size, in terms of total flow loading, is given in Equation 15 (Severin and Collins, 1992). For example, a 1-m belt with a distribution width $W = 80$ cm, operated at a belt speed of $s_b = 20$ cm/s at a flow of $Q_s + Q_p = 10\ 000\ cm^3/s$ is to be modeled with a filter cloth of area $A = 78.5\ cm^2$. The target test volume, V_o, is 490 cm^3. It should be noted that one sample volume is representative of a loci of other belt speeds and flow rates. In all tests performed for this paper, sample ranges were chosen to reflect application rates that are typical for full-scale belt press operation.

$$V_o = \frac{(Q_S + Q_p)A}{Ws_b} \qquad (15)$$

Test Methods

The method developed to mimic the drainage on a belt press originated as a simple drainage test by which conditioned sludge is poured onto a filter cloth of known area (Poduska and Collins, 1980; Baskerville *et al.*, 1978). Similar methods were used herein. There are three basic steps to the gravity drainage test: sludge conditioning (polymer addition and blending); pouring onto the drainage belt materials; and filtrate collection and timing.

Most of the polymers used in this study are classified as emulsion type having high molecular weights, and high-charge densities. The preparation method for a 0.4% by volume solution was as follows. Tap water (500 mL) was measured into a disposable polyethylene quart container. Polymer was drawn into a syringe. A high-speed, handheld kitchen blender was used to agitate the water as 2.0 mL of polymer was applied to the container. The blending was continued for 20 to 25 seconds. The polymer additions were prepared 20 minutes to 1 hour before use.

Sludge was conditioned by pouring the desired quantity of sludge into a disposable quart container. Dilution water was added to vary the solids content for some of the tests at this point. Polymer was then added by syringe as the sludge was being poured into a second quarter container. The sludge was flocculated by gently pouring the mixture between the two containers a number of times. The degree of mixing was constant for any series of tests, but varied from 6 to 10 pours based on visual observations of the particular sludge. Other researchers prefer to mechanically stir the conditioner into the sludge under more highly controlled conditions (Novak and Bandak, 1989).

Gravity drainage of conditioned sludge was performed by pouring the conditioned mixture onto belt filter material and measuring the filtrate volume as a function of time. A laboratory timer with a sound feature was used to facilitate the timing procedure. Filtrate volumes were marked on a graduated cylinder at the predetermined time intervals (5, 10, 15, 20, 30, 45, and 60 seconds). The volumes were then subsequently recorded from the marked cylinder.

The filter apparatus was the same as described by Poduska and Collins (1980) and similar to units described elsewhere

Table 2—Replicate drainage tests (filtrate volumes); large municipality sludge.

Time, s	Test 1a V_f, mL	Test 1b V_f, mL	Test 1c V_f, mL	Test 1d V_f, mL	Test 1e V_f, mL	Average V_f, mL	Std Dev. V_f, mL
0	0	0	0	0	0	0	0
5	109	118	116	122	116	116.2	4.7
10	152	160	150	161	158	156.2	4.9
15	172	184	169	182	176	176.6	6.4
20	187	190	187	195	193	190.4	3.6
30	198	198	202	203	202	200.6	2.4
45	211	215	214	212	211	212.6	1.8
60	216	224	218	220	216	218.8	3.3

Table 3a—Truncated look-up table as a function of V_f/V_F *KABT* (example $\gamma = 0.00595$); see Equation 9 and Table 3b.

KABt	V_f/V_f	*KABt*	V_f/V_F	*KABt*	V_f/V_F	*KABt*	V_f/V_F
0.256	0.55	0.470	0.68	0.866	0.81	1.899	0.940 0
0.268	0.56	0.492	0.69	0.910	0.82	2.073	0.950 0
0.282	0.57	0.515	0.70	0.958	0.83	2.288	0.960 0
0.295	0.58	0.539	0.71	1.009	0.84	2.569	0.970 0
0.310	0.59	0.565	0.72	1.064	0.85	2.968	0.980 0
0.325	0.60	0.591	0.73	1.124	0.86	3.253	0.985 0
0.340	0.61	0.619	0.74	1.189	0.87	3.657	0.990 0
0.356	0.62	0.649	0.75	1.260	0.88	4.014	0.993 0
0.373	0.63	0.680	0.76	1.337	0.89	4.865	0.997 0
0.391	0.64	0.713	0.77	1.424	0.90	5.972	0.999 0
0.409	0.65	0.748	0.78	1.520	0.91	6.671	0.999 5
0.429	0.66	0.766	0.785	1.629	0.92	6.896	0.999 6
0.449	0.67	0.785	0.79	1.754	0.93	7.186	0.999 7
		0.824	0.80			8.294	0.999 9

Table 3b—Example of calculation methods for finding best-fit parameters textile mill waste activated sludge. Belt filter cloth was Scandiafelt FE 3366, with a permeability (*κ/l*) of 5.6 sec^{-1}; see Table 4, Test 4, for test conditions.

Column A: Time, s	B: Average V_f, mL	C: Filtrate fraction V_f/V_F	D: *KABt*	E: Look-up fraction V_f/V_F	F: Calculate filtrate V_f, mL	G: Deviation squared, mL^2
0	0	0.000	0.000	0.000	0.0	0.0
5	130	0.660	0.422	0.656	129.3	0.5
10	163.5	0.830	0.843	0.804	158.5	25.3
15	173	0.878	1.265	0.880	173.4	0.2
20	180.5	0.916	1.686	0.924	182.1	2.6
30	187.5	0.951	2.529	0.968	190.8	10.8
45	195.5	0.992	3.794	0.991	195.3	0.0
60	199	1.010	5.058	0.998	196.6	5.7

Best-fit *KAB*, s^{-1}	0.0843 3	Sum of squares of deviation, mL^2	45.2
Best-fit final filter volume *VF*, mL	197.1	Standard error, mL	2.5
Best-fit final cake volume	302.9		
Best-fit solution of *B* mL^{-1}	0.008 375		
Best-fit *KA*, mL/s	10.066		
Relative resistance γ	0.005 95		

(Baskerville *et al.,* 1978; Neogen Inc., 1993). The apparatus consisted of a 4 × 2-in. polyvinylchloride (PVC) pipe bushing and a 4-in. PVC connector adapted to hold a 10-cm diameter belt filter cloth ($A = 78.5\ \text{cm}^2$) over a funnel and graduated cylinder. The filter cloth was rinsed in a weak chlorine bleach solution and then flushed with water after each test.

All tests were repeated between two and five times to diminish the magnitude of measurement errors. Some variations in measurement are to be expected. Table 2 is an example of a test condition repeated five times. In this example, 350-mL samples of raw sludge having a solids content of 5.15% from a large municipal wastewater treatment plant were each conditioned with 49 mL of polymer to produce total initial volumes of 399 mL. The polymer was a Secodyne product received as 5% dry weight in water and then diluted 30 mL into 1 000 mL of water. The total polymer dose was approximately 4.1 g/kg or 8.2 lb/ton dry sludge. The data in Table 2 show that the test methods used were able to recreate volumetric measurements between tests to within 4% (1 standard deviation) of the average at any time. Usually the most difficult number to reproduce was the 5-second sample point.

Curve Fitting Method

A common method for determining kinetic parameters from experimental data is to find the model solution that best matches the experimental data by using a standard criteria, such as the minimum sum of squares of deviation. One method of solving Equation 9 is by iteration on the dependent variable, *KAB*. In addition to being tedious, this method is prone to sensitivity problems as the value of V_f/V_F approaches 1. Another option is to invert the analysis and test model solutions for the independent variable (time) against experimental times that best fit the observed dependent variable (V_f/V_F). This method has the drawback that the asymptomatic nature of the curve of V_f/V_F versus *t* creates large apparent errors at long drainage times. Small differences in the drainage fraction create large swings in predicted drainage times.

A third technique, and the method used for analysis of data

Table 4—Analysis of sludge drainage from four sludge sources.

Sludge type	Municipality, 60% primary 40% activated	Water treatment alum, silt	Plastics, fiber waste activated	Textile mill waste activated
Sludge concentration %TS	5.15	2.57	2.46	3.36
Sludge volume, mL	350	300	300	440
Polymer volume, mL	49	8.5	22	60
Initial total volume, mL	399	308.5	322	500
Polymer concentration, %	0.15	0.4	0.4	0.4
Polymer dose, g/kg	4.1	4.4	11.9	16.2
Polymer type	Secodyne	Stockhaeusen Praestol A3040L	Stockhaeusen 260 FL	American Cyanamid Magnifloc 2081
Drainage belt	IFC 6308	Scandiafelt FE 3366	Scandiafelt FE 3366	Scandiafelt FE 3366
Time, s	Test 1, V_f, mL	Test 2, V_f, mL	Test 3, V_f, mL	Test 4, V_f, mL
0	0.0	0.0	0.0	0.0
5	116.2	02.5	114.0	130.0
10	156.2	130.5	151.0	163.5
15	176.6	140.5	170.5	173.0
20	190.4	146.5	186.0	180.5
30	200.6	150.5	196.0	187.5
45	212.6	156.0	202.5	195.5
60	218.8	158.0	207.0	199.0
Best-fit *KAB*, s^{-1}	0.05	0.094 7	0.057	0.084 5
Best-fit γ	0.020 4	0.008	0.006 67	0.005 98
Best-fit *B*	0.010 11	0.012 97	0.013 7	0.008 37
Best-fit final filter volume V_f, mL	218.2	157	210.5	197.7
Sum of squares of deviation, mL^2	59.1	29.5	67.4	42.9
Standard error, mL	2.9	2.1	3.1	2.5

in this paper, involves the tabulation of a large number of solutions for *KABt* as a function of the drainage fraction, V_f/V_F. The analyses presented in the results section were based on step sizes of $\Delta\ V_f/V_F = 0.002$ from $V_f/V_F = 0$ to $V_f/V_F = 0.99$ and points including $V_f/V_F = 0.999$, 0.999 9 and 0.999 99. Calculations were performed with the use of a personal computer spreadsheet software (Microsoft Excel 4.0 and 5.0) using a function called LOOKUP, whereby a target datum is compared with a column of precalculated numbers. Upon finding a match, a number from a third column is returned to the calculation. By this method the spreadsheet model was directed to look up the value of V_f/V_F for a given value of *KABt,* where *t* is the time of an observation and *KAB* is an input value.

An example of a method for determining the best-fit kinetic parameters for a test series is given in Tables 3a and b. The example test set was created by conditioning 440 mL of textile mill waste activated sludge treated with 60 mL of American Cyanamid Magnifloc 2081 cationic polymer and draining the conditioned sample on a 78.5-cm^2 filter made of Scandiafelt 3366 filter cloth. The average result from two replicates were

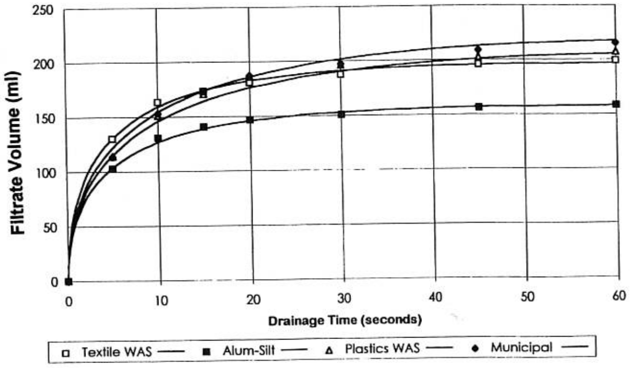

Figure 3—Examples of gravity drainage results from four sludges.

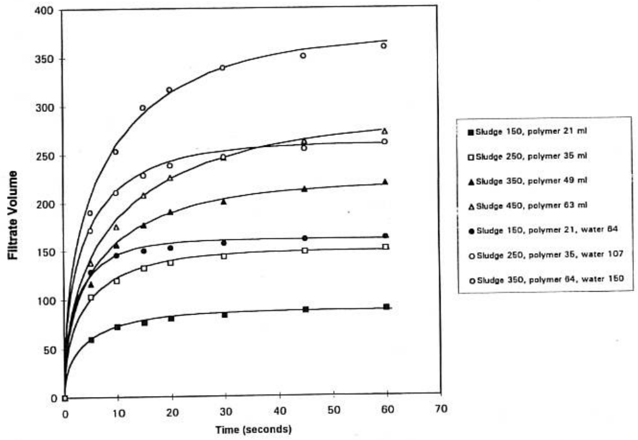

Figure 4—Effect of dilution and sample size on filtrate recovery with municipal sludge.

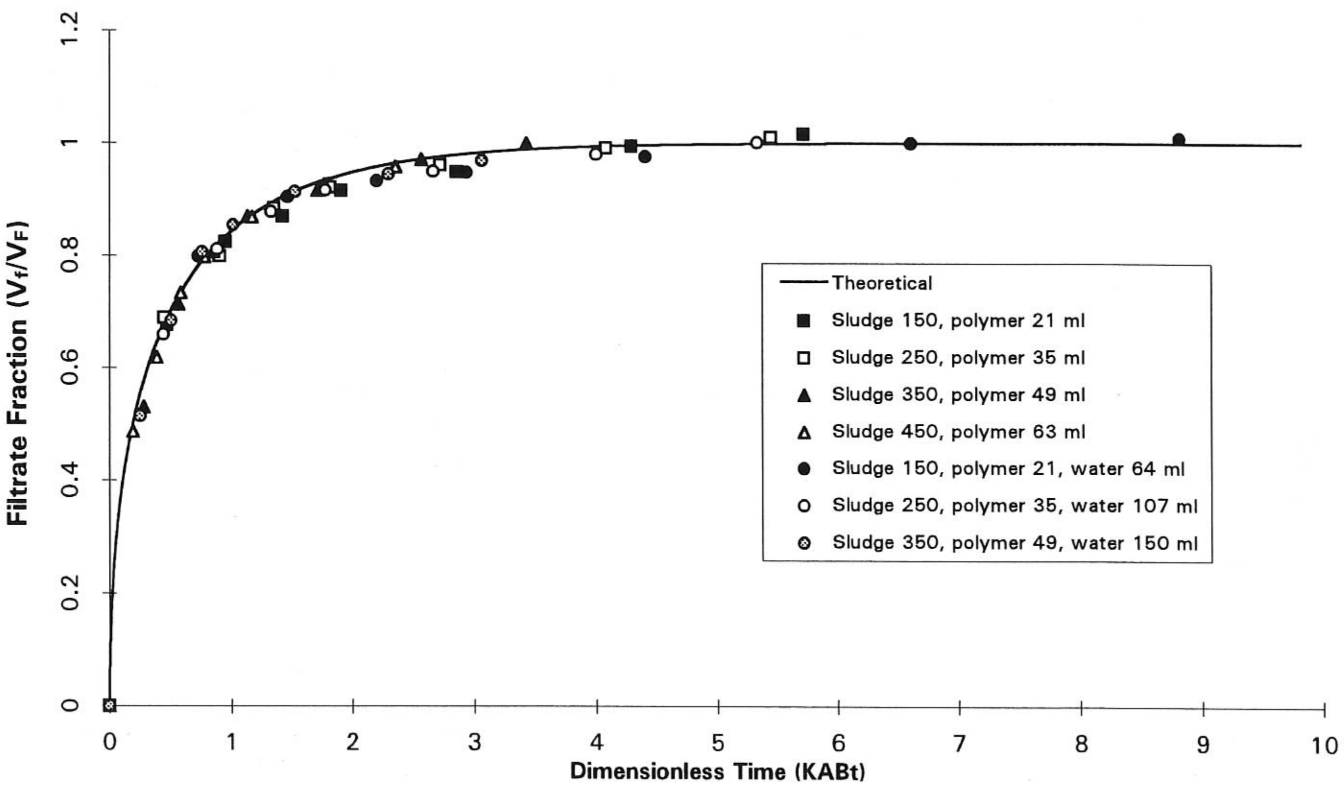

Figure 5—Fit of municipal sludge dewatering data to the dimensional model.

analyzed. Fitting the kinetic parameters involves comparison of tabulated values of V_f/V_F against tabulated values of *KABt*. Specific instructions are as follows with reference to an example LOOK-UP Table, Table 3a, and an example calculation tabulation, Table 3b.

1. Create a LOOK-UP table of V_f/V_F and corresponding *KABt*. The values of V_f/V_F range from 0.0 to 0.999 9 and are preset. The *KABt* values for the table are calculated for computed values of V_f/V_F and γ by use of Equation 9. The value of γ is linked to a constant filter resistance, κ/ℓ, the filter area *A*, the drainage coefficient *K*, an estimate of the ultimate cake volume, V_∞. Values for V_∞, *KAB*, and *B* are obtained in Steps 3, 4, and 5. Table 3 is an example of a LOOK-UP table calculated for the specific textile mill sludge and a γ value of 0.005 95.

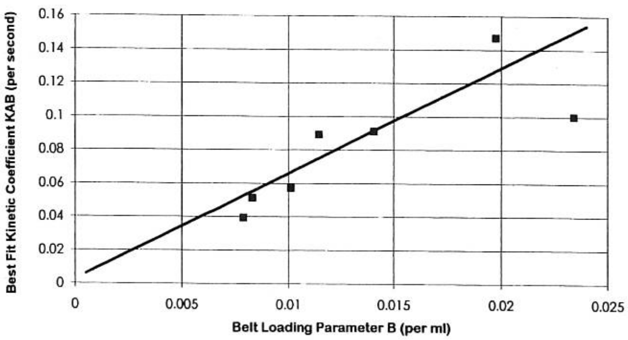

Figure 6—Fit of *KAB* versus *B* for municipal dewatering.

2. Tabulate the experimental values of time (Column A), and average filtrate for the elapsed time (Column B).

3. Guess a value of V_F and calculate the filtrate fraction recovered at each time (Column C). The value of V_F usually exceeds the 60-second filtrate value by a few milliliters.

4. Guess a value for *KAB*, multiply it by each sample time, and tabulate *KABt* for each sample time (Column D).

5. Calculate $V_\infty = V_o - V_F$, γ from Equation 12, and B from Equation 11 by using the values of V_F and *KAB* from instructions 3 and 4 above, recalling that $KA = KAB/B$. Recalculate the value of *KABt* in the LOOK-UP table (Equation 9) by using the set of values of V_f/V_F and γ that generated this step.

6. Find the values of *KABt* from Column D in the LOOK-UP table and return the corresponding value of V_f/V_F to Column E. This is the predicted value of V_f/V_F for this set of *KAB* and V_F, and γ.

7. Calculate the predicted filtrate volume as the product of the predicted value of V_f/V_F and V_F. Tabulate the result in Column F.

8. Subtract the predicted filtrate volume in Column F from the observed volume in Column B for each sample time. Square each result and tabulate it in Column G.

9. Sum the values in Column G. This is the sum of squares of deviations. Divide the sum of squares of deviation by the number of sample times and take the square root. This is the point standard error (mL).

10. Reguess V_F (instruction 3), *KAB* (instruction 4), and continue instructions 5 to 10, selecting values of V_F and *KAB* to minimize the sum of squares of deviation (Column G).

11. Note that every change in *KAB* and V_F changes the value

Table 5—Effect of dilution and volume on fitted parameters.

Slude type: Municipal (large city), 60% primary plus 40% waste activated
Sludge concentration: 5.15% TS
Polymer type: Secodyne; received as 5% dry in water, diluted 30 mL into 1 000 mL
Polymer dose: 4.1 g/kg dry sludge
Belt type: IFC 6308

	Test 5	Test 6	Test 7	Test 8	Test 9	Test 10	Test 11
No. of tests averaged	2	5	5	2	2	2	2
Sludge volume, mL	150	250	350	450	150	250	350
Dilution water	0	0	0	0	64	107	150
Polymer volume, mL	21	35	49	63	21	35	49
Initial total volume V_o, mL	171	285	399	513	235	392	549
Best-fit *KAB*, s^{-1}	0.101	0.090 7	0.057 1	0.039 3	0.147	0.09	0.051 1
Best-fit final filter volume V_F, mL	86.7	149.6	219.2	283.7	161.5	261	371
Loading factor *B*, 1/mL	0.023 4	0.014 1	0.010 1	0.007 9	0.019 8	0.01	0.008 3
Sum of squares of deviation, mL^2	13.6	30.2	64.5	64.6	105.0	56.7	279
Sludge volume ratio	0.54	0.54	0.51	0.51	0.49	0.53	0.51
Standard error, mL	1.4	2.1	3	3	3.9	2.8	6.3

of *KABt* in the LOOK-UP table. The spreadsheet should reflect this flow of information.

Results and Analysis

The drainage model given by Equation 9 can be used to curve fit drainage data from many types of conditioned sludges. Table 4 is a summary of drainage rate tests using sludges from a variety of different sources including textile mill waste activated sludge, alum-silt sludge from a water treatment plant, waste activated sludge from plastics manufacture, and combined primary and waste activated sludge from a large municipality. The standard error, estimated as the square root of the quotient of

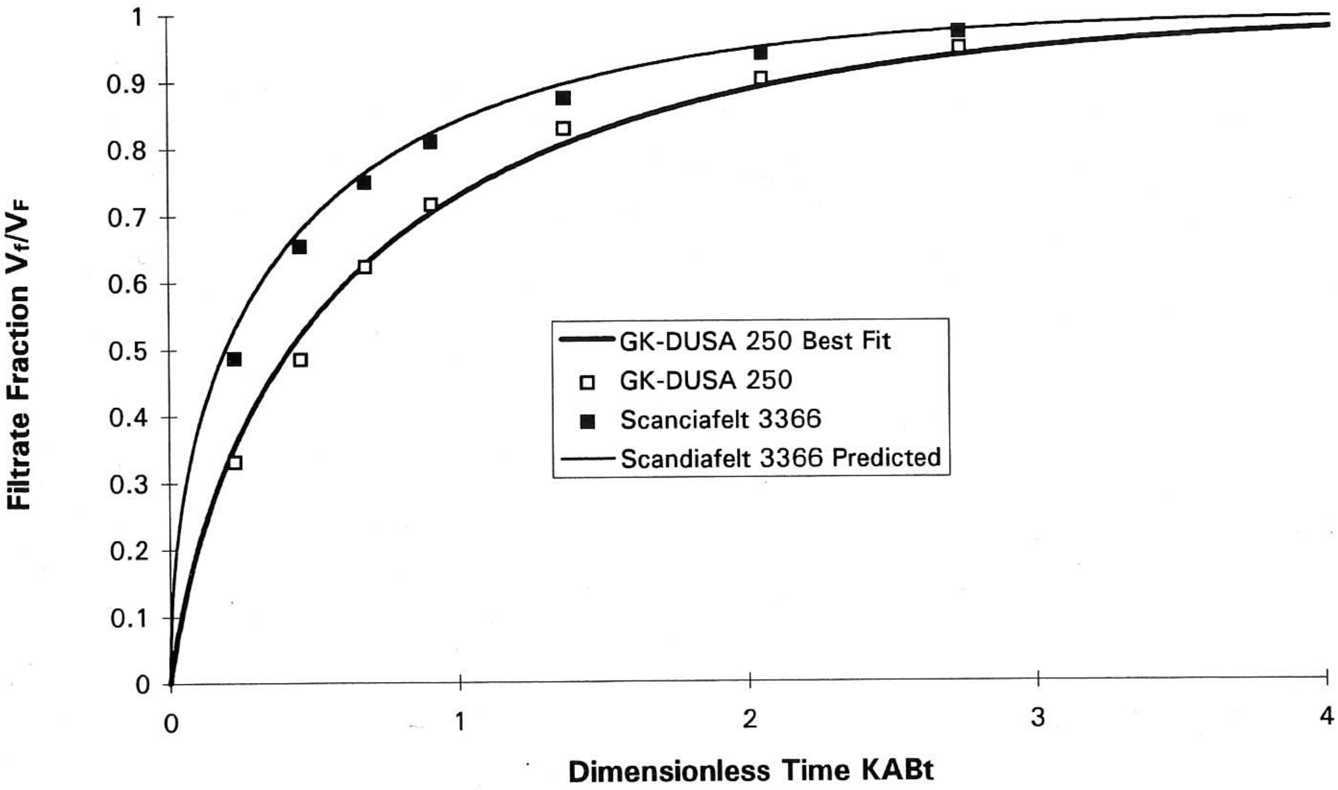

Figure 7—Example of effects of filter cloth resistance on drainage rate.

Table 6—Data for prediction of effect of cloth resistance; waste activated sludge from a small municipality.

Sludge concentration %TS	1.75	1.75
Sludge volume, mL	300	300
Dilution water	0	0
Polymer volume, mL	17	17
Initial total volume, mL	317	317
Polymer concentration, %	0.40	0.40
Polymer dose, g/kg dry solids	1.3	1.3
Polymer type	Stockhaeusen Praestol	
Drainage belt	GK-DUSKA 250 $\kappa/\ell = 0.75$	Scandiafelt 3366 $\kappa/\ell = 5.6$
Time, s	Average V_f, mL	Average V_f, mL
0	0	0
5	56.7	83
10	83	111.7
15	106.7	128.0
20	122.7	138.3
30	142.0	149.3
45	154.7	160.7
60	162.3	166.0
	Parameters fitted from above data	Fitted to parameter from Test 1
Best-fit *KAB*, per s	0.454	0.454
Best-fit final filter volume V_f, mL	172.2	172.2
Loading factor *B*, per mL	0.01271	0.01271
Resistance ratio γ	0.33	0.0044
Sum of squares of deviation, mL^2	76.2	125.4
Standard error, mL	3.3	4.3

the sum of squares of deviation divided by the number of sample points ($n = 7$) is low, from 2.1 to 2.9 mL per sample point. This is on the same order of magnitude as the standard deviation of the measured errors within multiple repeated samples (see Table 2). Figure 3 shows the fit of the model solution solved for the parameters *KAB* and V_F as outlined in Table 4.

Another feature of the model is the ability to correlate drainage rates caused by changes in sludge volume and solids concentration for a set of data collected with a constant polymer dose (g/kg dry solids). Seven sets of data were collected by using sludge from a large municipality wastewater treatment plant that produces a mixed sludge of approximately 5% total solids consisting of approximately 40% primary settled sludge and 60% waste activated sludge. Secodyne polymer and IFC 6308 filter cloth were used for these tests. The first four tests were performed with undiluted sludge and volumes of 150 to 350 mL. The final three tests were performed with sludge diluted approximately 42% with tap water, keeping the polymer dose (g/kg) constant on a dry weight basis. The approximate polymer dose was 4.1 g/kg. Figure 4 shows the raw data plotted as recovered filtrate volume versus time for the seven data sets. The plotted lines represent the best-fit model solution to the data. A summary of the conditions and fitted parameters for the seven data sets is given in Table 5.

One assumption used in the model derivation was that the ultimate cake volume is constant with respect to the initial slurry volume. For this to be true, then all data sets using equivalent concentrations of the same sludge should produce the same cake volume per volume of initial slurry. All diluted sludges should also produce cake volumes proportional to the initial sludge volume. This is true within a reasonable range of sampling error with a ratio of V_∞ to initial sludge volume ranging from 49% to 54% (see sludge volume ratios in Table 5) for the sets of data from the municipality. One test of the validity of the model is whether a wide range of data from a wide range of test volumes and highly diluted samples can be fitted to the model. Figure 5 shows that the data sets, independently fitted to the model, are normalized by the model. It may, therefore, be speculated that the model has utility in predicting the effects of belt speeds, sludge loading, and sludge concentration on drainage rates.

Another point previously discussed concerned the relative effect of γ on the shape of the curve of V_f/V_F versus *KABt* as summarized in Figure 2. It was stated that it was unlikely that a differentiation could be made between two tests with the same sludge unless the difference in γ between the runs exceeded 0.1. For the seven tests with the sludge from the large municipality, γ ranged from 0.016 to 0.075 with the maximum difference of 0.059 between any two tests. Figure 5 is a plot of V_f/V_F versus *KABt* for all seven sets of data from Table 5. The curve is calculated for $\gamma = 0.0$. As expected, the data all fit reasonably well to the model for $\gamma = 0.0$. As such, it should be expected that the drainage coefficient, *KA* should be similar for all test runs. The values of *KAB* plotted against the values of *B* for each data set are presented in Figure 6. The expected trend is observed with a fair fit to a slope of $KA = 6.0$ (cm^3/s) for six of the seven data sets. The outlier is Test 5 in Table 5. This test represents an undiluted sludge with an initial total volume of 171 mL. This volume may be too small to yield representative drainage data by these methods for this particular sludge and conditioning.

It was further speculated in the discussions of γ and Figure 2 that a change in belt material should yield an observable and predictable change in the shape of the curve of V_f/V_F versus $KABt$ if γ were increased significantly. Two data sets were collected to test this relationship. Both sets used the same waste activated sludge collected from a small municipality and the same polymer conditioning. Triplicate tests with 300 mL waste activated sludge at 1.75% total solids concentration were run for each fabric. Each sample was dosed with 17.0 mL Stockhaeusen Praestol polymer to yield a polymer dose of 1.3 g/kg. Set one was run with GK-DUSA 250 cfm cloth ($\kappa/\ell = 0.075$) while the second was run on Scandiafelt 3366 ($\kappa/\ell = 5.6$). The first set was analyzed to find the best-fit parameters KAB and V_F. The best-fit KAB was 0.454, and the best-fit V_F was 172.2 mL for a point standard error of 3.3 mL. The calculated value of γ was 0.33. The fitted parameters were then used to predict the curve of V_f/V_F versus $KABt$ for the second set using $\gamma = 0.044$, calculated from the value KA from the first data set and κ/ℓ for the Scandiafelt method. In Figure 7, a reasonable prediction was made between the two data sets, producing a point standard error of only 4.3 mL between the observed data and the prediction for the second set of data (Table 6).

Conclusions

- A mathematical model for the analysis of gravity drainage of conditioned sludge was derived from Darcy's equation of flow through porous media and assuming an average cake resistance throughout cake development. The model includes resistance parameters for the sludge cake and filter cloth. Two new parameters, dimensionless drainage time $KABt$ and resistance ratio γ, were introduced.
- Data from 12 test runs with sludge originating from five different sources were presented to verify that the model generally fits the shape of flocculent sludge drainage curves.
- The model allows for mathematically comparing the effects of sludge dilution for sludges treated equivalently with polymer on a dry weight basis. Data from seven tests, which includes tests with different initial volumes of diluted and undiluted sludge conditioned to the same polymer dose per dry weight of sludge, were correlated by the model.
- Data were presented to show that with similarly conditioned sludges it is possible to predict the effects of a change in belt materials on drainage dynamics.
- A simple means of measuring the cloth resistance was described. This test may be easily translated to the field to rapidly assess the condition of operating belts. A discussion of the degree of influence of cloth resistances was presented, which may allow for quantitative prediction of the effects of belt cleaning on drainage dynamics.
- The sludge drainage test method presented could be markedly improved by automation of the data collection. Refinements to the model or its interpretation could be made with the use of a better database.
- This paper represents a first attempt to reduce the complex belt press process into definitive components that may be analyzed by using laboratory and predictive methods. The model needs to be tested under field conditions. Future work is also needed to understand the influence of mechanical plows on drainage dynamics. The ultimate goal of the modeling is to predict Class 1 ''wet migration'' failure at the wedge zone of belt presses. More work is needed to apply the results of the present model to this critical process zone on full-scale presses.

Nomenclature

A = flow area, cm^2;
B = loading parameter $(V_F + V_\infty)/(V_F \cdot V_\infty)$, cm^{-3};
g = gravitational acceleration, cm/s^2;
γ = resistance ratio (dimensionless);
h_e = effective height of the available undrained slurry, cm;
h_o = height of water in clean water cloth resistance test;
K = cake permeability, cm/s;
κ = filter cloth permeability, cm/s;
ℓ = depth of filter cloth, cm;
L = depth of porous medium, cm;
P = available pressure, $dynes/cm^2$;
ρ = density of slurry above the formed cake, g/cm^3;
Q = water or sludge flow, cm^3/s;
Q_p = volumetric flow of polymer solution, cm^3/s;
Q_s = volumetric flow of sludge, cm^3/s;
R = cake resistance, cm^{-2};
r = filter cloth resistance, cm^{-2};
S = separation coefficient, V_∞/V_F;
s_b = drainage deck belt speed, cm/s;
V_c = volume of cake at time = t
V_f = volume of filtrate at time = t, cm^3;
V_F = volume of filtrate at time t = infinity, cm^3;
V_o = volume of sludge slurry at t = 0, cm^3;
V_∞ = volume of cake at t = infinity, cm^3;
W = belt press filter cloth width available for gravity drainage, cm; and
μ = filtrate viscosity, $dyne/cm^2 \cdot s$.

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